Grupa C 12.09.2011.

1. $$\begin{aligned} 4x+y&=5\\ 3x+2y&=5\\ 6x+2y+2\lambda&=\lambda^2 \end{aligned}$$

$$\left[\begin{array}{cc|c} 4 & 1 & 5\\ 3 & 2 & 5\\ 6 & 2 & \lambda^2-2\lambda \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 4 & 5\\ 2 & 3 & 5\\ 2 & 6 & \lambda^2-2\lambda \end{array}\right] \begin{array}{c} II_v-2\cdot I_v\\ \sim\\ III_v-2\cdot I_v \end{array}$$

$$\sim \left[\begin{array}{cc|c} 1 & 4 & 5\\ 0 & -5 & -5\\ 0 & -2 & \lambda^2-2\lambda-10 \end{array}\right] \quad \underset{\sim}{II_v}\cdot\left(-\tfrac{1}{5}\right)$$

$$\sim \left[\begin{array}{cc|c} 1 & 4 & 5\\ 0 & 1 & 1\\ 0 & -2 & \lambda^2-2\lambda-10 \end{array}\right] \quad \underset{\sim}{III_v+2\cdot II_v} \quad \begin{array}{c} \begin{array}{cc} y & x \end{array}\\ \left[\begin{array}{cc|c} 1 & 4 & 5\\ 0 & 1 & 1\\ 0 & 0 & \lambda^2-2\lambda-8 \end{array}\right] \end{array}$$

$$r(A)=2,\quad r(A|B)=\begin{cases} 2, & \text{ako je } \lambda=4 \lor \lambda=-2\\ 3, & \text{ako je } \lambda\neq 4 \land \lambda\neq -2 \end{cases}$$

$$\lambda^2-2\lambda-8=(\lambda-4)(\lambda+2)=0 \Rightarrow \lambda_1=4,\ \lambda_2=-2$$

Zaključak:

1° Ako je $\lambda\neq 4$ i $\lambda\neq -2$ sistem nema rješenje

2° Ako je $\lambda=4$ ili $\lambda=-2$, sistem ima jedinstveno rješenje, jer se svodi na:

$$\begin{aligned} y+4x&=5\\ \underline{x=1}& \end{aligned} \Rightarrow \begin{aligned} y+4&=5\\ x&=1 \end{aligned} \Rightarrow \begin{aligned} y&=1\\ x&=1 \end{aligned} \Rightarrow (1,1)\text{-rješenje}$$

2. $y = \ln \frac{x^2}{2-x}$

$\frac{x^2}{2-x} > 0 \Rightarrow x \neq 0 \wedge x < 2 \Rightarrow x \in (-\infty, 0) \cup (0, 2)$

- funkcija je ni parna ni neparna
- Nule: $\frac{x^2}{2-x} = 1 \Rightarrow x^2 = 2 - x$

$x^2 + x - 2 = 0 \Rightarrow x_1 = 1, \ x_2 = -2$

- Znak:

	$-\infty$	-2	0	1	2
y	$+$	0 $-$	$-$	0 $+$	

- V.A. $\lim_{x \to 0_-} \ln \frac{x^2}{2-x} = \ln 0_+ = -\infty$

$\lim_{x \to 0_+} \ln \frac{x^2}{2-x} = \ln 0_+ = -\infty$

vert. asympt. $x = 0$

$\lim_{x \to 2_-} \ln \frac{x^2}{2-x} = \ln \frac{4}{0_+} = \ln(+\infty) = +\infty$

$\Rightarrow$ vert. asimptota $x = 2$ (lijeva)

- H.A. $\lim_{x \to -\infty} \ln \frac{x^2}{2-x} \overset{/:x}{\underset{/:x}{=}} \lim_{x \to -\infty} \ln \frac{x}{\frac{2}{x} - 1} = \ln(+\infty) = +\infty$

nema H.A.

- K.A. $k = \lim_{x \to -\infty} \frac{\ln \frac{x^2}{2-x}}{x} = \lim_{x \to -\infty} \frac{\ln x^2 - \ln(2-x)}{x} = \frac{+\infty}{+\infty}$

$\overset{L.P.}{=} \lim_{x \to -\infty} \frac{\frac{2x}{x^2} + \frac{1}{2-x}}{1} = \frac{0}{1} = 0 \Rightarrow$ nema K.A.

$$y' = \frac{2-x}{x^2} \cdot \frac{2x(2-x)+x^2}{(2-x)^2} = \frac{4x-2x^2+x^2}{x^2(2-x)}.$$

$$y' = \frac{4x-x^2}{x^2(2-x)} = \frac{x(4-x)}{x^2(2-x)}$$

$$y' = \frac{4-x}{x(2-x)}$$

$$y' = 0 \Rightarrow \boxed{x = 4}, \quad 4 \notin D.p.$$

	$-\infty$	0	2
y'	$-$		$+$
y	$\searrow$		$\nearrow$
		н.Д.	

$$y'' = \left(\frac{4-x}{2x-x^2}\right)' = \frac{-(2x-x^2)-(4-x)\cdot(2-2x)}{(2x-x^2)^2}$$

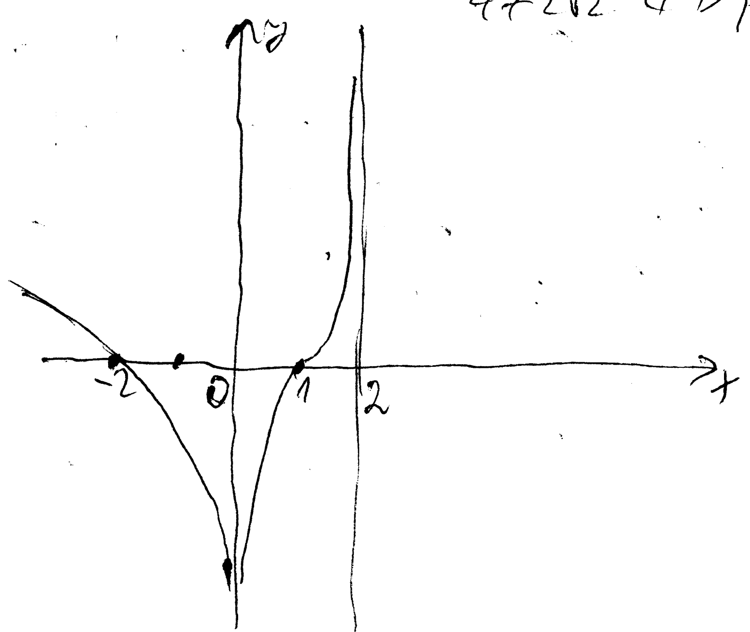
$$= \frac{\cancel{-2x}+x^2-8+8x\cancel{+2x}-2x^2}{(2x-x^2)^2}$$

$$= \frac{-x^2+8x-8}{(2x-x^2)^2} = \frac{-(x^2-8x+8)}{(2x-x^2)^2}$$

$$y'' = 0 \Rightarrow x^2-8x+8=0 \Rightarrow x_{1,2} = \frac{8 \pm 4\sqrt{2}}{2} = 4 \pm 2\sqrt{2}$$

$$4+2\sqrt{2} \notin D.p.$$

	$-\infty$	$4-2\sqrt{2}$	2
y''	$-$	0	$+$
y	$\cap$		$\cup$
		P.T.	



3 $$I = \int \frac{2x^4 - 2x^3 - x^2 + 2}{2x^3 - 4x^2 + 3x - 1}\,dx$$

$$(2x^4 - 2x^3 - x^2 + 2) : (2x^3 - 4x^2 + 3x - 1) = x + 1 + \frac{-2x+3}{2x^3 - 4x^2 + 3x - 1}$$

$$\underline{-2x^4 \mp 4x^3 \pm 3x^2 \mp x}$$

$$= \quad 2x^3 - 4x^2 + x + 2$$

$$\underline{-2x^3 \mp 4x^2 \pm 3x \mp 1}$$

$$= \quad = -2x + 3$$

$$I = \int \left(x + 1 + \frac{-2x+3}{2x^3 - 4x^2 + 3x - 1}\right)dx = \frac{x^2}{2} + x + \underbrace{\int \frac{-2x+3}{2x^3 - 4x^2 + 3x - 1}\,dx}_{I_1}$$

$$2x^3 - 4x^2 + 3x - 1 = 2x^3 - 2x^2 - 2x^2 + 2x + x - 1$$

$$= 2x^2(x-1) - 2x(x-1) + (x-1)$$

$$= (x-1)(2x^2 - 2x + 1)$$

$$\frac{-2x+3}{(x-1)(2x^2-2x+1)} = \frac{a}{x-1} + \frac{bx+c}{2x^2-2x+1}$$

$$-2x + 3 = a(2x^2 - 2x + 1) + (bx + c)(x - 1)$$

$$-2x + 3 = 2ax^2 - 2ax + a + bx^2 - bx + cx - c$$

uz x^2: $2a + b = 0 \Rightarrow b = -2a$

uz x: $-2a - b + c = -2 \quad \ldots (*)$

uz x^0: $\underline{a - c = 3} \Rightarrow c = a - 3$

$(*) \Rightarrow -2a + 2a + a - 3 = -2 \Rightarrow a = 1,\ b = -2,\ c = -2$

$$I_1 = \int \frac{1}{x-1}\,dx - 2\underbrace{\int \frac{x+1}{2x^2 - 2x + 1}\,dx}_{I_2} = \ln|x-1| - 2I_2$$

$$x + 1 = \alpha(4x - 2) + \beta \Rightarrow \alpha = \frac{1}{4},\ \beta = \frac{3}{2}$$

$$I_2 = \int \frac{\frac{1}{4}(4x-2)+\frac{3}{2}}{2x^2-2x+1}dx = \frac{1}{4}\int \frac{4x-2}{2x^2-2x+1}dx + \frac{3}{2}\underbrace{\int \frac{dx}{2x^2-2x+1}}_{I_3}$$

$$= \frac{1}{4}\ln(2x^2-2x+1) + \frac{3}{2} I_3 .$$

$$2x^2-2x+1 = 2(x^2-x+\frac{1}{2}) = 2[x^2-2\cdot x\cdot\frac{1}{2}+(\frac{1}{2})^2+\frac{1}{4}]$$

$$= 2[(x-\frac{1}{2})^2+\frac{1}{4}]$$

$$I_3 = \frac{1}{2}\int\frac{dx}{(x-\frac{1}{2})^2+\frac{1}{4}} = \left| \begin{matrix} x-\frac{1}{2}=t \\ dx=dt \end{matrix} \right|$$

$$= \frac{1}{2}\int \frac{dt}{t^2+\frac{1}{4}} = \frac{1}{2}\cdot 2\,\mathrm{arctg}\,2t + C$$

$$= \mathrm{arctg}(2t)+C = \mathrm{arctg}(2x-1)+C$$

$$I = \frac{x^2}{2}+x+\ln|x-1| - \frac{1}{2}\ln(2x^2-2x+1) - 3\,\mathrm{arctg}(2x-1)+C$$

4. $$y' - \mathrm{tg}\,y = \frac{e^x}{\cos y}$$

$$y' - \frac{\sin y}{\cos y} = \frac{e^x}{\cos y} \quad /\cdot\cos y$$

$$\cos y\cdot y' - \sin y = e^x$$

$$\sin y = z = z(x) \Rightarrow \cos y\cdot y' = z'$$

$$z' - z = e^x$$

$$z = uv \Rightarrow z' = u'v + uv'$$

$$u'v + uv' - uv = e^x$$

$$u'v + u\underbrace{(v' - v)}_{=0} = e^x$$

$$v' = v \Rightarrow \frac{v'}{v} = 1 \quad / dt, \int$$

$$\int \frac{dv}{v} = \int dt \Rightarrow \ln|v| = x \Rightarrow \boxed{v = e^x}$$

$$u' \cdot e^x = e^x \Rightarrow u' = 1 \Rightarrow u = \int dt = x + C$$

$$z = uv$$

$$z = (x + C) \cdot e^x$$

Opće rješenje: $\boxed{\ln y = (x + C) \cdot e^x}$